

After bulky brane inflation

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Reheating or entropy production on the three-brane from a decaying bulk scalar field is studied in the brane-world picture of the Universe. It is shown that a significant amount of dark radiation is produced in this process unless only the modes which satisfy a specific relation are excited, so that subsequent entropy production within the brane is required in general before primordial nucleosynthesis.

PACS Numbers: 98.80.Cq;04.50.+h;11.25.Mj;12.10.-g OU-TAP-158

Any new theory of gravity and/or high energy physics must pass a number of cosmological tests, among which is implementation of cosmological inflation [1,2]. Successful inflation must fulfill three requirements, namely, sufficiently long quasi-exponential expansion driven by vacuum-like energy density such as a potential energy of a scalar field, termination of accelerated expansion associated with entropy production or reheating to set the initial state of the classical hot Big Bang cosmology well before the primordial nucleosynthesis [3], and generation of primordial fluctuations with desired amplitude and spectrum [4]. It is much more difficult to achieve the second element than the first in general, and the third one typically requires fine tuning of model parameters.

In this paper we consider reheating after inflation in the brane world picture of the Universe [5,6]. In this scenario, our Universe is described on the four-dimensional boundary (three-brane) of Z_2 -symmetric five-dimensional spacetime with a negative cosmological constant $\Lambda_5 \equiv -6k^2$, where k is a positive constant. This situation not only takes into account the spirit of Horava-Witten theory [7,8], but also recovers the Einstein gravity around the brane with positive tension [6,9,10]. Much work has been done on brane-world cosmology [11–13] including inflationary brane solutions [14–17].

We assume the five-dimensional Einstein gravity with a negative cosmological constant Λ_5 and a three-brane at the fifth coordinate $w = 0$ about which the spacetime is Z_2 symmetric. We write the metric near the brane in the following form in terms of the Gaussian normal coordinate.

$$ds_5^2 = g_{AB}dx^A dx^B = -N^2(t, w)dt^2 + Q^2(t, w)a^2(t) (dx^2 + dy^2 + dz^2) + dw^2 \equiv q_{\mu\nu}dx^\mu dx^\nu + dw^2, \quad (1)$$

where capital Latin indices run 0,1,2,3, and 5 while Greek indices from 0 to 3. We take $N = Q = 1$ on the brane $w = 0$. One can write down functional forms of $N(t, w)$ and $Q(t, w)$ explicitly as

$$Q^2(t, w) = \cosh(2kw) + \frac{1}{2}k^{-2}H^2 (\cosh(2kw) - 1) - \sqrt{1 + k^{-2}H^2 + Ca^{-4}} \sinh(2k|w|),$$

$$N^2(t, w) = Q^{-2}(t, w) \left[\cosh(2kw) + \frac{1}{2}k^{-2} (H^2 + \dot{H}) (\cosh(2kw) - 1) - \frac{1 + \frac{1}{2}k^{-2} (2H^2 + \dot{H})}{\sqrt{1 + k^{-2}H^2 + Ca^{-4}}} \sinh(2k|w|) \right]^2, \quad (2)$$

in the case the bulk is in a vacuum state with a negative cosmological constant Λ_5 , where C is an integration constant [13]. In this solution the induced metric on the brane is nothing but the spatially flat Robertson-Walker metric with the scale factor $a(t)$.

The evolution equation on the brane in this case is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_5^4 \sigma}{18} \rho_{\text{tot}} + \frac{\Lambda_4}{3} + \frac{\kappa_5^4}{36} \rho_{\text{tot}}^2 - \frac{k^2 C}{a^4}, \quad (3)$$

$$\Lambda_4 \equiv \frac{1}{2} \left(\Lambda_5 + \frac{\kappa_5^2}{6} \sigma^2 \right), \quad (4)$$

where κ_5^2 is the five dimensional gravitational constant related with the five dimensional reduced Planck scale, M_5 , by $\kappa_5^2 = M_5^{-3}$. Here σ is the brane tension, ρ_{tot} is the total energy density on the brane, and the last term of (3) represents the so-called dark radiation with C being an integration constant [10,12,13]. In order to recover the standard Friedmann equation with a vanishing cosmological constant at low energy scales, we require $\sigma = 6k/\kappa_5^2$ and $\kappa_4^2 = \kappa_5^4 \sigma / 6 = \kappa_5^2 k$, where κ_4^2 is the four dimensional gravitational constant related with the four dimensional reduced Planck scale, M_4 , as $\kappa_4^2 = M_4^{-2}$. We therefore find $M_4^2 = M_5^3/k$. That is, if we take $k = M_4$, all the fundamental scales in the theory take the same value, $k = M_4 = M_5$. Note that k also sets the scale above which the nonstandard term quadratic in ρ_{tot} is effective in (3). We assume that k is much larger than the scale of inflation so that such quadratic corrections are negligible.

We consider the case inflation is driven by a bulk scalar field ϕ with a five-dimensional potential $V[\phi]$ [16,17] and study the evolution of ϕ after brane inflation, because reheating is expected to proceed in the same way as in four dimensional theory if the inflaton lives only on the brane [15]. Since ϕ is homogenized in three space as a result of inflation, it depends only on t

and w . We consider a situation that ϕ rapidly oscillates around $\phi = 0$ and assume that $V[\phi]$ is expressed as $V[\phi] = m^2\phi^2/2$. Then the Klein-Gordon equation reads

$$\square_5\phi(w, t) - V'[\phi(w, t)] = \frac{1}{\sqrt{-g}}\partial_t(\sqrt{-g}g^{00}\dot{\phi}) + \frac{1}{\sqrt{-g}}\partial_w(\sqrt{-g}\phi_{,w}) - V'[\phi] = 0, \quad (5)$$

where a dot denotes time differentiation. In order to express energy release of ϕ we introduce the following dissipation term phenomenologically in (5).

$$\square_5\phi(w, t) - V'[\phi(w, t)] = \frac{\Gamma_D}{2k}\delta(w)\frac{1}{N}\dot{\phi} + \Gamma_B\frac{1}{N}\dot{\phi}. \quad (6)$$

Here Γ_D and Γ_B represent energy release to the brane and to the entire space, respectively. The denominator in the right-hand-side is introduced on dimensional grounds.

From (5) and (6) together with the Z_2 symmetry, we find

$$\phi_{,w}^+ = -\phi_{,w}^- = \frac{\Gamma_D}{4k}\dot{\phi}(0, t), \quad (7)$$

where superscripts $+$ and $-$ imply values at $w \rightarrow +0$ and -0 , respectively.

The divergence of the energy-momentum tensor of the scalar field,

$$T_{MN}^{(\phi)} = \phi_{,M}\phi_{,N} - g_{MN}\left(\frac{1}{2}g^{PQ}\phi_{,P}\phi_{,Q} + V[\phi]\right), \quad (8)$$

reads,

$$T_{A;C}^{(\phi)C} = \{\square_5\phi(w, t) - V'[\phi(w, t)]\}\phi_{,A} = \left[\frac{\Gamma_D}{2k}\delta(w)\frac{1}{N}\dot{\phi} + \Gamma_B\frac{1}{N}\dot{\phi}\right]\phi_{,A}. \quad (9)$$

Integrating $A = 0$ component of (9) from $w = -\epsilon$ to $w = +\epsilon$ near the brane, we find (7) from the zeroth order in ϵ and

$$\frac{\partial\rho_\phi(0, t)}{\partial t} = -(3H + \Gamma_B)\dot{\phi}^2(0, t) - J_\phi(0, t), \quad (10)$$

with

$$\rho_\phi \equiv \frac{1}{2}\dot{\phi}^2 + V[\phi], \quad J_\phi \equiv -\frac{\dot{\phi}}{\sqrt{-g}}\partial_w(\sqrt{-g}\phi_{,w}), \quad (11)$$

from the terms proportional to ϵ . Thus the energy dissipated by the Γ_D term on the brane is entirely compensated by the energy flow onto the brane.

Next we study how the energy released from ϕ affects evolution of our brane Universe by analyzing gravitational field equations [10,17]. In the present situation the total energy momentum tensor including the contribution of bulk cosmological constant reads

$$T_{MN} = -\kappa_5^{-2}\Lambda_5 g_{MN} + T_{MN}^{(\phi)} + S_{MN}\delta(w), \quad (12)$$

where S_{MN} is the stress tensor on the brane. Its nonvanishing components can be further decomposed as

$$S_{\mu\nu} = -\sigma q_{\mu\nu} + \tau_{\mu\nu}. \quad (13)$$

Here $\tau_{\mu\nu}$ represents energy momentum tensor of the radiation fields produced by decay of ϕ and it is of the form $\tau_\nu^\mu = \text{diag}(-\rho_r, p_r, p_r, p_r)$ with $p_r = \rho_r/3$.

In terms of the unit vector $n_M = (0, 0, 0, 0, 1)$ normal to the brane, the extrinsic curvature of a $w = \text{constant}$ hypersurface is given by $K_{MN} = q_M^P q_N^Q n_{Q;P}$ with $q_{MN} = g_{MN} - n_M n_N$. Then from the Codazzi equation and the five dimensional Einstein equation, we find

$$D_\nu K_\mu^\nu - D_\mu K = \kappa_5^2 T_{MN} n^N q_\mu^M = \kappa_5^2 T_{\mu w} = \kappa_5^2 \dot{\phi}\phi_{,w}\delta_\mu^0, \quad (14)$$

where D_ν stands for the four dimensional covariant derivative with respect to the metric $q_{\mu\nu}$. The above equation reads

$$D_\nu K_0^{\nu+} - D_0 K^+ = \kappa_5^2 \frac{\Gamma_D}{4k}\dot{\phi}^2(0, t), \quad (15)$$

near the brane, $w \rightarrow +0$.

From the junction condition and the Z_2 -symmetry, on the other hand, we find

$$K_{\mu\nu}^+ = -\frac{\kappa_5^2}{2}\left(S_{\mu\nu} - \frac{1}{3}q_{\mu\nu}S\right), \quad (16)$$

therefore

$$D_\nu K_\mu^{\nu+} - D_\mu K^+ = -\frac{\kappa_5^2}{2} D_\nu S_\mu^\nu = -\frac{\kappa_5^2}{2} D_\nu \tau_\mu^\nu. \quad (17)$$

Combining (15) and (17), we obtain

$$D_\nu \tau_\mu^\nu = -\frac{\Gamma_D}{2k} \dot{\phi}^2 \delta_\mu^0, \quad (18)$$

namely,

$$\frac{\partial \rho_r}{\partial t} = -3H(\rho_r + p_r) + \frac{\Gamma_D}{2k} \dot{\phi}^2 = -4H\rho_r + \frac{\Gamma_D}{2k} \dot{\phi}^2, \quad (19)$$

on the brane. Thus we find only the dissipation term proportional to Γ_D with the delta function is effective to reheat the brane. This equation has the same form as the reheating in perturbation theory after conventional inflation in four dimensional theory [18].

On the other hand, the four dimensional Einstein tensor, $G_\mu^{(4)\nu}$, satisfies the following equality on the brane [17],

$$G_\mu^{(4)\nu} = \kappa_4^2 (T_\mu^{(s)\nu} + \tau_\mu^\nu) + \kappa_5^4 \pi_\mu^\nu - E_\mu^\nu, \quad (20)$$

with

$$T_\mu^{(s)\nu} \equiv \frac{1}{6k} \left[4q^{\nu\zeta} \phi_{,\mu} \phi_{,\zeta} + \left(\frac{3}{2} \phi_{,w}^2 - \frac{5}{2} q^{\xi\zeta} \phi_{,\xi} \phi_{,\zeta} - \frac{3}{2} m^2 \phi^2 \right) q_\mu^\nu \right]. \quad (21)$$

Here π_μ^ν represents terms quadratic in τ_α^β which are higher order in $\rho_r/(kM_4)^2$ and are consistently neglected in our analysis. $E_\mu^\nu \equiv C_{\mu w}^{w\nu}$ is a component of the five dimensional Weyl tensor C_{PQ}^{MN} , which is the origin of the dark radiation [13].

Now we write down the four dimensional Bianchi identity,

$$D_\nu G_\mu^{(4)\nu} = 0 = \kappa_4^2 (D_\nu T_\mu^{(s)\nu} + D_\nu \tau_\mu^\nu) - D_\nu E_\mu^\nu, \quad (22)$$

to yield

$$D_\nu E_0^\nu = -\frac{\kappa_4^2}{2k} \frac{\partial}{\partial t} \left(\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \phi_{,w}^2 + \frac{1}{2} m^2 \phi^2 \right) - \frac{2\kappa_4^2 H}{k} \dot{\phi}^2 - \frac{\kappa_4^2}{2k} \Gamma_D \dot{\phi}^2. \quad (23)$$

Note that $\phi_{,w}^2 = \Gamma_D^2 \dot{\phi}^2 / (16k^2)$ on the brane and is negligibly small compared with $\dot{\phi}^2$, since we expect the dissipation rate Γ_D is at most comparable to the scale of inflation and hence much smaller than the scale k . With this approximation the quantity in the parenthesis in (23) may be replaced by $\rho_\phi(0, t)$. We also find $D_\nu E_0^\nu = \partial_0 E_0^0 + 4H E_0^0$ because E_μ^ν is traceless.

As a result we obtain the following set of evolution equations in the brane universe $w = 0$ in terms of $\varphi(t) \equiv \phi(0, t)/\sqrt{2k}$.

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa_4^2}{3} (\rho_\varphi + \rho_r + \varepsilon), \quad \rho_\varphi \equiv \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} m^2 \varphi^2 = \frac{\rho_\phi}{2k}, \quad \varepsilon \equiv \frac{E_0^0}{\kappa_4^2}, \quad (24)$$

$$\frac{\partial \rho_\varphi}{\partial t} = -(3H + \Gamma_B) \dot{\varphi}^2 - J_\varphi, \quad J_\varphi \equiv \frac{J_\phi}{2k}, \quad (25)$$

$$\frac{\partial \rho_r}{\partial t} = -4H\rho_r + \Gamma_D \dot{\varphi}^2, \quad (26)$$

$$\frac{\partial \varepsilon}{\partial t} = -4H\varepsilon - (H + \Gamma_D - \Gamma_B) \dot{\varphi}^2 + J_\varphi, \quad (27)$$

where we have used (10).

Here J_φ represents energy flow of the scalar field into the bulk and we cannot calculate it unless we solve the field equation in the bulk. It is, however, formidable to do so in the realistic situation that the bulk metric is influenced by the bulk scalar field which would be the case in brane inflation driven by a bulk scalar field. In this case the metric would be even more complicated than (2). Hence in order to obtain some insights on the form of J_φ we must employ several approximations and find a bulk scalar solution.

Let us assume the bulk metric is governed by Λ_5 and further neglect terms suppressed by k^{-1} and Ca^{-4} in (2). Then the metric reads

$$ds_5^2 = -e^{-2k|w|} dt^2 + e^{-2k|w|} a^2(t) (dx^2 + dy^2 + dz^2) + dw^2, \quad (28)$$

and field equations (5) and (6) become more tractable. In the time scale of the field oscillation the dissipation terms are unimportant because they are perturbatively small quantities by assumption. So let us estimate J_φ by solving (5) instead of (6) first. The solution of (5) under the metric (28) is easily found in the same way as Goldberger and Wise [19] as

$$\phi(t, w) = \sum_n c_n T_n(t) Y_n(w) + H.C. \quad (29)$$

with

$$T_n(t) \cong a^{-\frac{3}{2}}(t) e^{-im_n t}, \quad (30)$$

$$Y_n(w) = e^{2k|w|} \left[J_\nu \left(\frac{m_n}{k} e^{k|w|} \right) + b_n N_\nu \left(\frac{m_n}{k} e^{k|w|} \right) \right], \quad \nu = 2\sqrt{1 + \frac{m^2}{4k^2}} \cong 2 + \frac{m^2}{4k^2}, \quad (31)$$

under the assumption that the field oscillates rapidly in cosmic expansion time scale. Here m_n is a separation constant which may take continuous values in the present case with a single brane, and b_n is a constant determined by the boundary condition, $\phi_{,w} = 0$, at $w = 0$ as

$$b_n = \left[2J_\nu \left(\frac{m_n}{k} \right) + \frac{m_n}{k} J'_\nu \left(\frac{m_n}{k} \right) \right] \left[2N_\nu \left(\frac{m_n}{k} \right) + \frac{m_n}{k} N'_\nu \left(\frac{m_n}{k} \right) \right]^{-1}. \quad (32)$$

If we incorporate the effect of dissipation on the boundary condition, (7), it is modified to

$$b_n \cong \left[\left(2 + \frac{im_n \Gamma_D}{2k^2} \right) J_\nu \left(\frac{m_n}{k} \right) + \frac{m_n}{k} J'_\nu \left(\frac{m_n}{k} \right) \right] \left[\left(2 + \frac{im_n \Gamma_D}{2k^2} \right) N_\nu \left(\frac{m_n}{k} \right) + \frac{m_n}{k} N'_\nu \left(\frac{m_n}{k} \right) \right]^{-1}, \quad (33)$$

where use has been made of $\dot{T}_n(t) \cong -im_n T_n(t)$.

Hereafter let us assume only a single oscillation mode exists. This is a natural assumption because any higher mode is expected to decay earlier and the final stage of reheating would be dominated by the lowest mode that has been excited. Then we find

$$J_\varphi = (m_n^2 - m^2) \varphi \dot{\varphi}. \quad (34)$$

Since φ oscillates rapidly in the expansion time scale let us average the right-hand-side of evolution equations (25)–(27) over an oscillation period. Using $\overline{\varphi^2}(t) = m_n^2 \overline{\varphi^2}(t)$ and $\overline{\varphi \dot{\varphi}}(t) = -(3H + \Gamma_B) \overline{\varphi^2}(t)/2$, we obtain the following set of evolution equations in the brane universe $w = 0$ where a bar denotes average over the oscillation period.

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 \cong \frac{\kappa_4^2}{3} (\rho_\varphi + \rho_r + \varepsilon), \quad (35)$$

$$\frac{\partial \rho_\varphi}{\partial t} = -\frac{1}{2} (3H + \Gamma_B) (m^2 + m_n^2) \overline{\varphi^2}, \quad (36)$$

$$\frac{\partial \rho_r}{\partial t} = -4H \rho_r + \Gamma_D m_n^2 \overline{\varphi^2}, \quad (37)$$

$$\frac{\partial \varepsilon}{\partial t} = -4H \varepsilon - (\Gamma_D + H - \Gamma_B) m_n^2 \overline{\varphi^2} + \frac{1}{2} (3H + \Gamma_B) (m^2 - m_n^2) \overline{\varphi^2}, \quad (38)$$

with

$$\overline{\varphi^2}(t) \equiv \overline{\varphi_i^2} \left(\frac{a(t)}{a(t_i)} \right)^{-3} e^{-\Gamma_B(t-t_i)}. \quad (39)$$

We assume that both ρ_r and ε are vanishingly small at the end of inflation when we set the initial condition, because they rapidly redshift during inflation and very little creation is expected in that period [17]. We identify the reheating epoch with the time when ρ_φ becomes smaller than ρ_r . From (39) this occurs at $t \simeq H^{-1} \simeq \Gamma_B^{-1}$, so that the dominant creation takes place during $H \gtrsim \Gamma_B$. Here we qualitatively analyze the system for various values of m_n .

Case A: $m_n \geq m$

In this case, the creation terms of dark radiation, $-(\Gamma_D + H - \Gamma_B) m_n^2 \overline{\varphi^2} + \frac{1}{2} (3H + \Gamma_B) (m^2 - m_n^2) \overline{\varphi^2}$ are negative and their magnitude is larger than the creation term of radiation, $\Gamma_D m_n^2 \overline{\varphi^2}$ during the important period $H \gtrsim \Gamma_B$. Hence more dark radiation is created than ordinary radiation in magnitude. Then ρ_r and ε tend to cancel each other and the higher-order terms of the Friedmann equation, which we have neglected so far, would play an important role in the subsequent evolution of the three brane. This means that we do not recover standard cosmology on the brane after inflation.

Case B: $m_n \ll m$

This possibility is not excluded in the brane-world scenario with non-factorizable geometry unlike in the case of ordinary Kalza-Klein compactification with factorizable metric [19]. In this case the last term of (38) is dominant and we find more dark radiation than ordinary radiation unless Γ_B is extremely small with $\Gamma_B/\Gamma_D < m_n^2/m^2 \ll 1$.

Case C: $m_n \lesssim m$

This is the most delicate case and the final amount of dark radiation can be either positive or negative depending on the details of the model parameters. But in order to achieve successful primordial nucleosynthesis, the amount of extra radiation-like matter is severely constrained [3]. In order to have sufficiently small ε compared with ρ_r after reheating without resorting to subsequent entropy production within the brane, the magnitude of creation terms of ε should be vanishingly small at the reheating epoch $H \simeq \Gamma_B$. That is, only the mode that satisfies the inequality

$$|2\Gamma_B(m^2 - m_n^2) - \Gamma_D m_n^2| \ll \Gamma_D m_n^2, \quad (40)$$

should be present.

We thus find a specific relation (40) should be satisfied for the graceful exit of brane inflation driven by a bulk scalar field ϕ . In case it is not satisfied, we must introduce some other mechanisms of entropy production within the brane before primordial nucleosynthesis which imposes stringent constraints on the exotic energy density and the expansion law at that era [3]. One way is to assume that ϕ predominantly decays into massive particles whose energy density redshifts less rapidly and dominates over ε soon. If these particles decay into radiation before nucleosynthesis creating an appropriate amount of baryon asymmetry, we may recover the standard cosmology. Another way is to consider second inflation which is driven by a scalar field confined on the brane [15] to dilute ε .

In summary we have studied entropy production on the three-brane from a decaying bulk scalar field ϕ by introducing dissipation terms to its equation of motion phenomenologically. We have shown that the so-called dark radiation is significantly produced at the same time unless the inequality (40) is satisfied. Although we have analyzed only the case with a specific form of the dissipation, we expect our conclusion is generic and applicable to other forms of dissipation, too, because it is essentially an outcome of the four dimensional Bianchi identity (22). We therefore conclude that in the brane-world picture of the Universe, it is likely that the dominant part of the entropy we observe today originates within the brane rather than in the bulk.

The authors are grateful to M. Sasaki and T. Tanaka for useful communications. The work of JY was partially supported by the Monbukagakusho Grant-in-Aid, Priority Area ‘‘Supersymmetry and Unified Theory of Elementary Particles’’(#707) and the Monbukagakusho Grant-in-Aid for Scientific Research Nos. 11740146 and 13640285.

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